Finite Element Circuit Based Modeling for Computing Electromagnetic Forces in Synchronous Reluctance Machine Rotor

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This work demonstrates a novel finite element based modeling for computing the electromagnetic forces in a synchronous reluctance motor with different eccentricities of the rotor. Here, the model interpolates data from a look-up table created in advance by combining a numerical and an analytical approach where the relationship between the stator currents and flux linkages are identified with static finite element analysis. The state-space based model is capable enough like a non-linear time stepping finite element model to predict the electromagnetic forces on the rotor with different sets of uneven magnetic flux density across the airgap, even at much lower computational burden.

***Index Terms*—Additional winding, finite-element methods, induction machines, numerical models, state space methods, synchronous reluctance motor.**

I. Introduction

In the control of electrical machines, the traditional d-q model takes into account only the fundamental spatial permeance variation, and neglects magnetic saturation. To contemplate the machine behavior with more accurate torque characteristics, vibrations, a better model other than the aforementioned method is required. Finite element (FE) based methods are able to calculate more accurate characteristics of the machine behavior by considering all the wave harmonics and thus quite helpful for a feedback based control system. But the main hindrance in FE based simulation is the computation time taken by this approach.

A numerical-analytical model where the relationship between the stator currents and flux linkages are identified with static finite element analysis can address the aforesaid issue [1]. An eccentric rotor results in the production of the electromagnetic forces on the rotor which can be calculated by the time discretized two dimensional (2-D) FE analysis [2]. In view of controlling the rotor vibration, it is very important to know the electromagnetic forces on rotor at different eccentricities, and at different operational points [3]. With FE circuit based modeling, the control system can be designed accurately like the time discretized 2-D FE analysis to mitigate the rotor vibration.

In the scope of the present work, there are primarily two objectives. Firstly, calculating the electromagnetic forces on the rotor for a set of different eccentricities with a novel FE circuit based modeling and compare with the non-linear time stepping FE analysis. Secondly, calculating the electromagnetic forces on the rotor by implementing an additional force actuator winding in the stator slots. With the additional winding, it is possible to vary the magnetic flux density around the airgap resulting in the production of electromagnetic forces on the rotor [4][5].

In this work, a synchronous reluctance machine (SynRM) has been analyzed. A SynRM is rugged in structures, easy to manufacture, and most importantly, in compare to an induction machine of same size and power rating, a SynRM can show better efficiency [6]. In this paper, the SynRM is rated at 15 kW, and has two pole pairs. The main winding generates a four pole flux to produce the required torque, while the additional winding produces a two pole flux to create the unbalance magnetic flux density in the airgap and generates electromagnetic forces on the rotor as shown in Fig. 1.

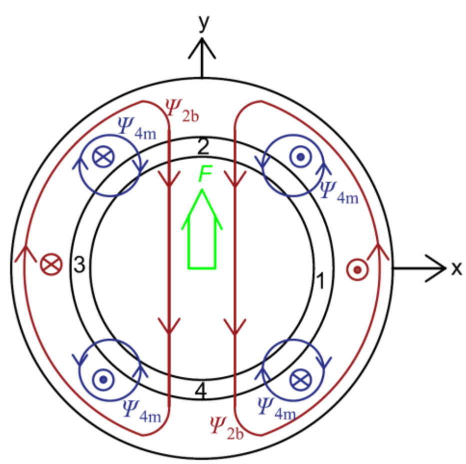

Fig. 1. Creation of unbalanced magnetic pull with additional winding [5].

II. FE Based Model Approach

In this section, the FE circuit based model is described. The voltage equation for SynRM is given by,

$$\boldsymbol{u} = R_s \boldsymbol{i} + \frac{d\boldsymbol{\psi}}{dt} \tag{1}$$

where $\boldsymbol{u}$, $\boldsymbol{i}$, and $\boldsymbol{\psi} = [\psi_x\ \psi_y]^T$ denote the voltage, current and flux linkages space vectors respectively in the stator [xy] frame of reference, and R_s is the resistance of stator phase winding. The precalculated FE solution can be used in state space model to replace the inductance functions. Using current as a state variable with rotor angle α_r and rotor displacement $\boldsymbol{d} = [d_x \; d_y]^T$, (1) can be re-written as

$$\boldsymbol{u} = R_s \boldsymbol{i} + \frac{\partial \boldsymbol{\psi}(\boldsymbol{i}, \alpha_r, \boldsymbol{d})}{\partial \boldsymbol{i}} \frac{d\boldsymbol{i}}{dt} + \omega_r \frac{\partial \boldsymbol{\psi}(\boldsymbol{i}, \alpha_r, \boldsymbol{d})}{\partial \alpha_r} + \frac{\partial \boldsymbol{\psi}(\boldsymbol{i}, \alpha_r, \boldsymbol{d})}{\partial \boldsymbol{d}} \boldsymbol{v} \quad (2)$$

where $\omega_r = d\alpha_r/dt$ and $\boldsymbol{v} = d\boldsymbol{d}/dt$. The advantage of selecting the current as the state variable is the readiness of implementing it in static FE analysis. The FE circuit based modeling has been implemented in the MATLAB/Simulink environment by deriving the three elements (assuming symmetry) of the Jacobian matrix $\partial\boldsymbol{\psi}/\partial\boldsymbol{i}$, $\partial\boldsymbol{\psi}/\partial\boldsymbol{d}$, the vector $\partial\boldsymbol{\psi}/\partial\alpha_r$ and the torque from static FE solutions and storing them in look-up table (LUT) [7]. Notably, the current in this model only refers to the main winding.

To create the multi-layer array for the LUT, nested loop simulations have been performed with multi-parameter input, and the result for multi-parameter output is stored in it. Without the additional winding, torque and electromagnetic forces are calculated from the LUT as,

$$\boldsymbol{\psi} = \boldsymbol{\psi}_{FE}(\boldsymbol{i}, \alpha_r, \boldsymbol{d}) \quad (3)$$

$$T = T_{FE}(\boldsymbol{i}, \alpha_r, \boldsymbol{d}) \quad (4)$$

$$\boldsymbol{F} = \boldsymbol{F}_{FE}(\boldsymbol{i}, \alpha_r, \boldsymbol{d}) \quad (5)$$

where T is the electromagnetic torque and $\boldsymbol{F} = [F_x \; F_y]^T$ is the force on the rotor according to Fig. 2(a). The finite element mesh of the studied machine is shown in Fig. 2(b).

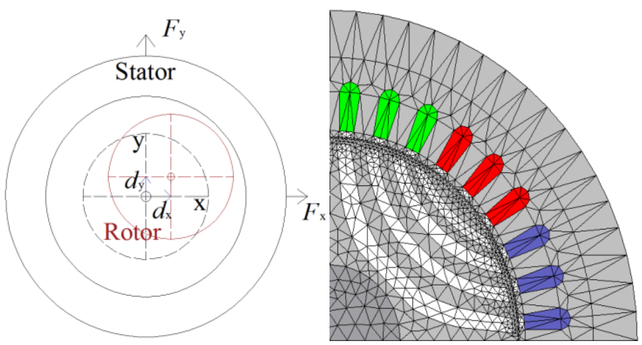

Fig. 2. (a) Rotor eccentricity leading to electromagnetic force generation and (b) FE mesh of the studied machine.

The spline block in the simulation is used for the linear interpolation. The simulation is performed with the variable step solver *ode45* with maximum time step size being limited to 80 steps per one supply period. The FE model has been simulated with fixed step size of 80 steps per one supply period using trapezoidal time integration.

III. Results and Discussions

The different forces in the x and y directions for 14% dynamic eccentricity are computed from time stepping FE simulation and FE circuit based model which have been presented in Fig. 3. F_{x1}, F_{x2} are the forces in the x direction, and F_{y1}, F_{y2} are the forces in the y direction for FE time stepping method and FE circuit based model respectively. The supply voltage is 300 V rms with star connection.

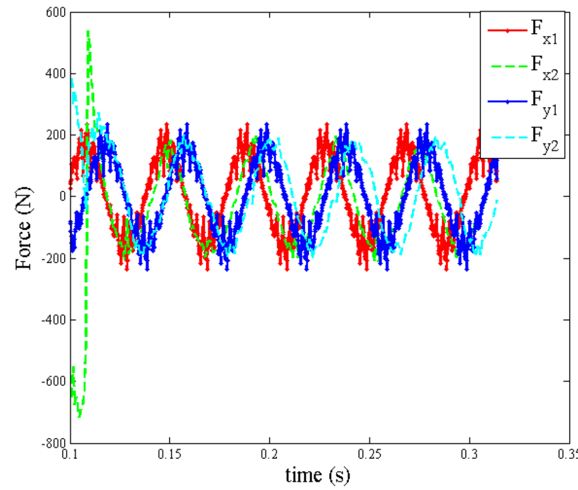

Fig. 3. Comparison of force calculation from time stepping FE simulation and FE circuit based model at 300 V rms voltage and 14 % dynamic eccentricity.

It can be seen from Fig. 3, that the forces from the FE circuit based model match closely with non-linear FE time stepping simulation. The model is validated by comparing the values of the torque and the electromagnetic forces on the rotor in the x and y directions with the non-linear time stepping FE simulation. In the complete paper, more detailed results of the above analysis will be presented. Also, the electromagnetic force computation will be presented with the additional winding and the results will be compared with the non-linear time stepping FE simulation.

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References

[1] M. Mohr, O. Biro and F. Diwoky, "Consideration of rotor eccentricity effects in a multi body dynamics simulation using a finite element based circuit model approach," *in Proc. ICEM*, pp. 1523-1528, Sept 2014.

[2] A. Arkkio, B. R. Nepal and A. Sinervo, "Electro mechanical interaction in a synchronous reluctance machine," *in Proc. SPEEDAM*, pp. 501-506, June 2010.

[3] M. Michon, R. C. Holehouse, K. Atallah and G. Johnstone, "Effect of rotor eccentricity in large synchronous machines," *IEEE Trans. Magn.*, vol. 50, no. 11, pp. 1-4, Nov 2014.

[4] A. Chiba, M. Rahman and T. Fukao, "Radial force in a bearingless reluctance motor", *IEEE Trans. Magn.*, vol. 27, no. 2, pp. 786-791, March 1991.

[5] M. Takemoto, K. Yoshida, N. Itasaka, Y. Tanaka, A. Chiba and T. Fukao, "Synchronous reluctance type bearingless motors with multi-flux barriers," *IEEE Power Convers. Conf.*, pp. 1559-1564, April 2007.

[6] S. J. Mun, Y. H. Cho and J. H. Lee, "Optimum design of synchronous reluctance motors based on torque/volume using finite-element method and sequential unconstrained minimization technique," *IEEE Trans. Magn.*, vol. 44, no. 11, pp. 4143-4146, Nov 2008.

[7] P. Rasilo, M. A. Lemesle, A. Belahcen, A. Arkkio and M. Hinkkanen "Comparison of finite-element-based state-space models for PM synchronous machines," *IEEE Trans. Energy Convers.*, vol. 29, no. 2, pp. 535-543, June 2014.